Mathematical & Computational Theory of Graph Dynamical Systems

Graph Dynamical Systems as a Framework for Analysis and Simulation of Complex Systems:

Mathematical Aspects and Theory

Asymptotic behavior of complex systems:
- Fixed points of permutation SDS are independent of the update schedule.
- SDS induced by the Boolean function Nor have fixed points.
- SDS induced by threshold functions only have fixed points as attractors.

Classification of limit cycle structure in asynchronous systems:
- The structural diversity of long-term behavior in asynchronous systems is governed by k-equivalence. We have constructed a bound for the number of long term behaviors that is possible for a fixed graph and fixed functions when the permutation composition sequence is varied.
- For the circle graph on vertices, there are only n identities long term limit cycles.

Generalization of standard threshold systems
- Applications: a large range of phenomena can be captured and described as threshold systems (epidemics, belief propagation, rumors, fads).
- Classical, networked threshold models use binary states and a common threshold k for the transitions from 0 to 1 and from 1 to 0. Although these models are well-understood mathematically, they do always suffice for modeling.
- We have generalized the class of threshold systems in several important ways. Examples include all of the following:
  - Bi-threshold systems are extension of classical threshold systems where the two transitions 0 to 1 and 1 to 0 have separate thresholds k⁺ (up threshold) and k⁻ (down threshold).
  - Dynamic threshold systems have thresholds that change with the dynamics and can capture phenomena with increased/ decreased susceptibility, tolerance or immunity.
  - Multi-state bi-threshold systems combines the first extension above with a generalized state set \( \{0,1,2,\ldots\} \).

Theorem: Synchronous bi-threshold systems only have fixed points and 2-cycles as limit cycles regardless of the choice of k⁺ and k⁻.

Algorithmic and Computational Aspects and Theory

Reachability: Starting from a configuration \( \mathbb{C} \) can \( S \) reach \( T \) in less than \( r \) steps? Starting from a given configuration, will the SDS \( S \) ever reach a fixed point? What sort of fixed points?

Application - Viral Marketing: choose a small subset of users to initially introduce a product, so that the maximum number of people adopt it, assuming the popularity spreads by a diffusion process.

Computational Complexity Results:
- Computing dynamical properties of general GDS not known in polynomial time.
- Efficient algorithms often possible if the underlying graph has a tree-like structure.

The InterSim Framework

InterSim is a distributed, configurable, and extensible, with plugable interaction models that represent various functions.

InterSim scales to 100 million nodes and 2 billion edges.

Epidemiology

Markets

Distributed Computing

Research theme: Derive qualitative and quantitative information about a complex system/model based on known properties (e.g. graph, functions, update scheme) without exhaustive, brute-force computations. Examples:
- How does the graph, vertex functions and/or update scheme affect the global dynamics?
- Is the system robust with respect to perturbations such as edge alterations or changes to vertex states?
- For a given level of resolution (i.e. equivalence measure), how can the neural networks be characterized? Since most system properties are only known locally, the approach typically follows the local-to-global paradigm.

Graph and influence domain:

Composed Map: \( F = F_4(1,5) \oplus F_4(1,2) \oplus F_4(2,5) \oplus F_4(5,1) \)

Asynchronous threshold systems undergo a fundamental change that may have long limit cycles. When \( k \leq 2 \), only fixed points are possible.

Consequence: Theory gives insight into system behavior, parameter interdependencies, scheduling sensitivity, and offers support for system validation of corresponding interactionist models.

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